

## Percolation within Percolation

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The following programme was written in a macro language of Matlab. It was used in works leading to a Ph.D. thesis (Tiyapan, 2004). Percolation within percolation is the theme of that thesis.

### Regular 3-dimension lattice

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1 % trr.m, regular 3-d tessellation, (c) Kit Tiyapan, 16 Dec 2002
2 qn=size(q,1);
3 in2n=size(in2,1);
4 in3n=size(in3,1);
5 in5n=size(in5,1);
6 p=[];
7 p{1,1}=[o;o]';
8 p{1,2}=sz;
9 r=[];
10 s=[];
11 t=[];
12 for i=1:sz,
13     r(1,i)=dx(m(i));
14     s(1,i)=dy(n(i));
15     t(1,i)=dz(z(i));
16 end
17 p{1,3}=r';
18 p{1,4}=s';
19 p{1,5}=t';
20 iin=size(ii,1);
21 tmp=ones(iin,1);
22 tma=sparse(tmp,ii(:,1),tmp,1,sz);
23 tmp=[];
24 tmb=[];
25 tmc=[];
26 tmd=[];
27 cnt=0;
28 for i=1:sz,
29     if (~tma(i))
30         cnt=cnt+1;
31         tmp=[tmp;i,cnt];
32         tmb=[tmb;r(i)];
33         tmc=[tmc;s(i)];
34         tmd=[tmd;t(i)];
35     end
36 end
37 p{2,1}=tmp;
38 p{2,2}=cnt;
39 p{2,3}=tmb;
40 p{2,4}=tmc;
41 p{2,5}=tmd;
42 iiii=size(iii,1);
43 tmp=ones(iiii,1);
44 tma=sparse(tmp,iii(:,1),tmp,1,sz);
45 tmp=[];

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46 tmb=[];
47 tmc=[];
48 tmd=[];
49 cnt=0;
50 for i=1:sz,
51     if(~tma(i))
52         cnt=cnt+1;
53         tmp=[tmp;i,cnt];
54         tmb=[tmb;r(i)];
55         tmc=[tmc;s(i)];
56         tmd=[tmd;t(i)];
57     end
58 end
59 p{3,1}=tmp;
60 p{3,2}=cnt;
61 p{3,3}=tmb;
62 p{3,4}=tmc;
63 p{3,5}=tmd;
64 iv=[ii,2*ones(iin,1)];
65 for i=1:iin,
66     tmp=0;
67     for j=1:iin,
68         if(~(iii(i,1)-ii(j,1)))
69             tmp=1;
70         end
71     end
72     if(~tmp)
73         iv=[iv;iii(i,:),3];
74     end
75 end
76 invn=size(iv,1);
77 tmp=ones(invn,1);
78 tma=sparse(tmp,iv(:,1),tmp,1,sz);
79 tmp=[];
80 tmb=[];
81 tmc=[];
82 tmd=[];
83 cnt=0;
84 for i=1:sz,
85     if(~tma(i))
86         cnt=cnt+1;
87         tmp=[tmp;i,cnt];
88         tmb=[tmb;r(i)];
89         tmc=[tmc;s(i)];
90         tmd=[tmd;t(i)];
91     end
92 end
93 p{4,1}=tmp;
94 p{4,2}=cnt;
95 p{4,3}=tmb;
96 p{4,4}=tmc;
97 p{4,5}=tmd;
98 vn=size(v,1);
99 tmp=ones(vn,1);
100 tma=sparse(tmp,v(:,1),tmp,1,sz);
101 tmp=[];
102 tmb=[];
103 tmc=[];
104 tmd=[];
105 cnt=0;
106 for i=1:sz,
107     if(~tma(i))
108         cnt=cnt+1;

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109     tmp=[tmp;i,cnt];
110     tmb=[tmb;r(i)];
111     tmc=[tmc;s(i)];
112     tmd=[tmd;t(i)];
113 end
114 end
115 p{5,1}=tmp;
116 p{5,2}=cnt;
117 p{5,3}=tmb;
118 p{5,4}=tmc;
119 p{5,5}=tmd;
120 vi=[ii,2*ones(iin,1)];
121 for i=1:vn,
122     tmp=0;
123     for j=1:iin,
124         if ~(v(i,1)-ii(j,1)))
125             tmp=1;
126         end
127     end
128     if (~tmp)
129         vi=[vi;v(i,:),5];
130     end
131 end
132 vin=size(vi,1);
133 tmp=ones(vin,1);
134 tma=sparse(tmp,vi(:,1),tmp,1,sz);
135 tmp=[];
136 tmb=[];
137 tmc=[];
138 tmd=[];
139 cnt=0;
140 for i=1:sz,
141     if (~tma(i))
142         cnt=cnt+1;
143         tmp=[tmp;i,cnt];
144         tmb=[tmb;r(i)];
145         tmc=[tmc;s(i)];
146         tmd=[tmd;t(i)];
147     end
148 end
149 p{6,1}=tmp;
150 p{6,2}=cnt;
151 p{6,3}=tmb;
152 p{6,4}=tmc;
153 p{6,5}=tmd;
154 vii=[iii,3*ones(iii,1)];
155 for i=1:vn,
156     tmp=0;
157     for j=1:iiin,
158         if ~(v(i,1)-iii(j,1)))
159             tmp=1;
160         end
161     end
162     if (~tmp)
163         vii=[vii;v(i,:),5];
164     end
165 end
166 viin=size(vii,1);
167 tmp=ones(viin,1);
168 tma=sparse(tmp,vii(:,1),tmp,1,sz);
169 tmp=[];
170 tmb=[];
171 tmc=[];

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172 tmd=[];
173 cnt=0;
174 for i=1:sz,
175     if(~tma(i))
176         cnt=cnt+1;
177         tmp=[tmp;i,cnt];
178         tmb=[tmb;r(i)];
179         tmc=[tmc;s(i)];
180         tmd=[tmd;t(i)];
181     end
182 end
183 p{7,1}=tmp;
184 p{7,2}=cnt;
185 p{7,3}=tmb;
186 p{7,4}=tmc;
187 p{7,5}=tmd;
188 viii=iv;
189 for i=1:vn,
190     tmp=0;
191     for j=1:ivn,
192         if(~(v(i,1)-iv(j,1)))
193             tmp=1;
194         end
195     end
196     if(~tmp)
197         viii=[viii;v(i,:),5];
198     end
199 end
200 viiin=size(viii,1);
201 tmp=ones(viiin,1);
202 tma=sparse(tmp,viii(:,1),tmp,1,sz);
203 tmp=[];
204 tmb=[];
205 tmc=[];
206 tmd=[];
207 cnt=0;
208 for i=1:sz,
209     if(~tma(i))
210         cnt=cnt+1;
211         tmp=[tmp;i,cnt];
212         tmb=[tmb;r(i)];
213         tmc=[tmc;s(i)];
214         tmd=[tmd;t(i)];
215     end
216 end
217 p{8,1}=tmp;
218 p{8,2}=cnt;
219 p{8,3}=tmb;
220 p{8,4}=tmc;
221 p{8,5}=tmd;
222 map=[];
223 tmp=ones(sz,1);
224 map{1,1,1}=sparse(p{1,1}(:,1),tmp,p{1,1}(:,2),sz,1);
225 cnt=p{1,2};
226 e=q;
227 V=[p{1,3},p{1,4},p{1,5}];
228 tmp=ones(p{2,2},1);
229 for i=2:nx,
230     map{i,1,1}=sparse(p{2,1}(:,1),tmp,cnt*tmp+p{2,1}(:,2),sz,1);
231     for k=1:iin,
232         map{i,1,1}(ii(k,1),1)=map{(i-1),1,1}(ii(k,2),1);
233     end
234     cnt=cnt+p{2,2};

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235 % +iin;
236 for k=1:qn,
237     e=[e;map{i,1,1}(q(k,1)),map{i,1,1}(q(k,2))];
238 end
239 for k=1:in2n,
240     e=[e;map{i,1,1}(in2(k,1)),map{(i-1),1,1}(in2(k,2))];
241 end
242 V=[V;(i-1)*dim1*tmp+p{2,3},p{2,4},p{2,5}];
243 end
244 tmp=ones(p{3,2},1);
245 for j=2:ny,
246     map{1,j,1}=sparse(p{3,1}(:,1),tmp,cnt*tmp+p{3,1}(:,2),sz,1);
247     for k=1:iiin,
248         map{1,j,1}(iii(k,1),1)=map{1,(j-1),1}(iii(k,2),1);
249     end
250     cnt=cnt+p{3,2};
251 % +iiin;
252 for k=1:qn,
253     e=[e;map{1,j,1}(q(k,1)),map{1,j,1}(q(k,2))];
254 end
255 for k=1:in3n,
256     e=[e;map{1,j,1}(in3(k,1)),map{1,(j-1),1}(in3(k,2))];
257 end
258 V=[V;p{3,3},(j-1)*dim2*tmp+p{3,4},p{3,5}];
259 end
260 tmp=ones(p{4,2},1);
261 for i=2:nx,
262     for j=2:ny,
263         map{i,j,1}=sparse(p{4,1}(:,1),tmp,cnt*tmp+p{4,1}(:,2),sz,1);
264         for k=1:ivn,
265             if(iv(k,3)==2)
266                 map{i,j,1}(iv(k,1),1)=map{(i-1),j,1}(iv(k,2),1);
267             else
268                 map{i,j,1}(iv(k,1),1)=map{i,(j-1),1}(iv(k,2),1);
269             end
270         end
271         cnt=cnt+p{4,2};
272 % +ivn;
273 for k=1:qn,
274     e=[e;map{i,j,1}(q(k,1)),map{i,j,1}(q(k,2))];
275 end
276 for k=1:in2n,
277     e=[e;map{i,j,1}(in2(k,1)),map{(i-1),j,1}(in2(k,2))];
278 end
279 for k=1:in3n,
280     e=[e;map{i,j,1}(in3(k,1)),map{i,(j-1),1}(in3(k,2))];
281 end
282 V=[V;(i-1)*dim1*tmp+p{4,3},(j-1)*dim2*tmp+p{4,4},p{4,5}];
283 end
284 end
285 tmp=ones(p{5,2},1);
286 for i=2:nz,
287     map{1,1,i}=sparse(p{5,1}(:,1),tmp,cnt*tmp+p{5,1}(:,2),sz,1);
288     for k=1:vn,
289         map{1,1,i}(v(k,1),1)=map{1,1,(i-1)}(v(k,2),1);
290     end
291     cnt=cnt+p{5,2};
292 for k=1:qn,
293     e=[e;map{1,1,i}(q(k,1)),map{1,1,i}(q(k,2))];
294 end
295 for k=1:in5n,
296     e=[e;map{1,1,i}(in5(k,1)),map{1,1,(i-1)}(in5(k,2))];
297 end

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298 V=[V;p{5,3},p{5,4} ,(i-1)*dim3*tmp+p{5,5}];
299 end
300 tmp=ones(p{6,2},1);
301 for i=2:nx,
302     for j=2:nz,
303         map{i,1,j}=sparse(p{6,1}(:,1),tmp,cnt*tmp+p{6,1}(:,2),sz,1);
304         for k=1:vin,
305             if ~(vi(k,3)-2))
306                 map{i,1,j}(vi(k,1),1)=map{(i-1),1,j}(vi(k,2),1);
307             else
308                 map{i,1,j}(vi(k,1),1)=map{i,1,(j-1)}(vi(k,2),1);
309             end
310         end
311         cnt=cnt+p{6,2};
312         for k=1:qn,
313             e=[e;map{i,1,j}(q(k,1)),map{i,1,j}(q(k,2))];
314         end
315         for k=1:in2n,
316             e=[e;map{i,1,j}(in2(k,1)),map{(i-1),1,j}(in2(k,2))];
317         end
318         for k=1:in5n,
319             e=[e;map{i,1,j}(in5(k,1)),map{i,1,(j-1)}(in5(k,2))];
320         end
321         V=[V;(i-1)*dim1*tmp+p{6,3},p{6,4} ,(j-1)*dim3*tmp+p{6,5}];
322     end
323 end
324 tmp=ones(p{7,2},1);
325 for i=2:ny,
326     for j=2:nz,
327         map{1,i,j}=sparse(p{7,1}(:,1),tmp,cnt*tmp+p{7,1}(:,2),sz,1);
328         for k=1:viiin,
329             if ~(vii(k,3)-3))
330                 map{1,i,j}(vii(k,1),1)=map{1,(i-1),j}(vii(k,2),1);
331             else
332                 map{1,i,j}(vii(k,1),1)=map{1,i,(j-1)}(vii(k,2),1);
333             end
334         end
335         cnt=cnt+p{7,2};
336         for k=1:qn,
337             e=[e;map{1,i,j}(q(k,1)),map{1,i,j}(q(k,2))];
338         end
339         for k=1:in3n,
340             e=[e;map{1,i,j}(in3(k,1)),map{1,(i-1),j}(in3(k,2))];
341         end
342         for k=1:in5n,
343             e=[e;map{1,i,j}(in5(k,1)),map{1,i,(j-1)}(in5(k,2))];
344         end
345         V=[V;p{7,3} ,(i-1)*dim2*tmp+p{7,4} ,(j-1)*dim3*tmp+p{7,5}];
346     end
347 end
348 tmp=ones(p{8,2},1);
349 for i=2:nx,
350     for j=2:ny,
351         for k=2:nz,
352             map{i,j,k}=sparse(p{8,1}(:,1),tmp,cnt*tmp+p{8,1}(:,2),sz,1);
353             for m=1:viiin,
354                 if ~(viii(m,3)-2))
355                     map{i,j,k}(viii(m,1),1)=map{(i-1),j,k}(viii(m,2),1);
356                 elseif ~(viii(m,3)-3))
357                     map{i,j,k}(viii(m,1),1)=map{i,(j-1),k}(viii(m,2),1);
358                 else
359                     map{i,j,k}(viii(m,1),1)=map{i,j,(k-1)}(viii(m,2),1);
360                 end

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361     end
362     cnt=cnt+p{8,2};
363     for m=1:qn,
364         e=[e;map{i,j,k}(q(m,1)),map{i,j,k}(q(m,2))];
365     end
366     for m=1:in2n,
367         e=[e;map{i,j,k}(in2(m,1)),map{(i-1),j,k}(in2(m,2))];
368     end
369     for m=1:in3n,
370         e=[e;map{i,j,k}(in3(m,1)),map{i,(j-1),k}(in3(m,2))];
371     end
372     for m=1:in5n,
373         e=[e;map{i,j,k}(in5(m,1)),map{i,j,(k-1)}(in5(m,2))];
374     end
375     V=[V;(i-1)*dim1*tmp+p{8,3},(j-1)*dim2*tmp+p{8,4},...
376         (k-1)*dim3*tmp+p{8,5}];
377     end
378 end
379 end
380 en=size(e,1);
381 Vn=size(V,1);
382 figure(1);
383 clf;
384 hold on;
385 for i=1:en,
386     plot3([V(e(i,1),1),V(e(i,2),1)], [V(e(i,1),2),V(e(i,2),2)],...
387         [V(e(i,1),3),V(e(i,2),3)]);
388 end
389 axis off;
390 axis equal;
391 clf;
392 hold on;
393 tms=sum(nemat,2);
394 tmp=[];
395 tma=17.2;
396 % 5x5x5 units
397 for i=1:en,
398     if(tms(i)<tma)
399         tmp=[tmp;e(i,:)];
400     end
401 end
402 tmn=size(tmp,1);
403 for i=1:tmn,
404     plot3([V(tmp(i,1),1),V(tmp(i,2),1)],...
405         [V(tmp(i,1),2),V(tmp(i,2),2)], [V(tmp(i,1),3),V(tmp(i,2),3)]);
406 end
407 axis off;
408 axis equal;
409 % for vertices
410 nvmat=sparse(Vn,Vn);
411 for i=1:en,
412     nvmat(e(i,1),e(i,2))=1;
413     nvmat(e(i,2),e(i,1))=1;
414 end
415 A=V;
416 N=Vn;
417 lmat=sparse(1,N);
418 umat=sparse(1,N);
419 LB=min(V(:,1));
420 UB=max(V(:,1));
421 rng=UB-LB;
422 LBv=.05*rng+LB;
423 UBv=UB-LBv;

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424 for i=1:Vn,
425     if(V(i,1)<LBv)
426         lmat(1,i)=1;
427     end
428     if(V(i,1)>UBv)
429         umat(1,i)=1;
430     end
431 end
432 nmat=nvmat;
433 Blocked=randperm(Vn);
434 [pc,cord,tsries]=perc(N,lmat,umat,nmat);
435 % for edges
436 evm=sparse(en,Vn);
437 for i=1:en,
438     evm(i,e(i,1))=1;
439     evm(i,e(i,2))=1;
440 end
441 nemat=sparse(en,en);
442 for i=1:Vn,
443     tmp=find(evm(:,i));
444     tmn=size(tmp,1);
445     for j=1:(tmn-1),
446         for k=(j+1):tmn,
447             nemat(tmp(j),tmp(k))=1;
448             nemat(tmp(k),tmp(j))=1;
449         end
450     end
451 end
452 A=e;
453 N=en;
454 lmat=sparse(1,N);
455 umat=sparse(1,N);
456 for i=1:N,
457     if((V(A(i,1),1)<=LBv) | (V(A(i,2),1)<=LBv))
458         lmat(1,i)=1;
459     elseif((V(A(i,1),1)>=UBv) | (V(A(i,2),1)>=UBv))
460         umat(1,i)=1;
461     end
462 end
463 nmat=nemat;
464 Blocked=randperm(N);
465 [pc,cord,tsries]=perc(N,lmat,umat,nmat);
466 % ccp
467 clear all;
468 sz=9;
469 nx=5;
470 ny=5;
471 nz=5;
472 r=1;
473 tmp=sqrt(2)*r;
474 dx=[0,2*r,4*r];
475 dy=[0,2*r,4*r];
476 dz=[0,tmp,2*tmp];
477 dim1=max(dx);
478 dim2=max(dy);
479 dim3=max(dz);
480 q=[1,2;1,3;1,5;1,6;2,5;3,5;4,5;5,6;5,7;5,8;5,9];
481 m=[1,3,1,3,2,1,3,1,3];
482 n=[1,1,3,3,2,1,1,3,3];
483 z=[1,1,1,1,2,3,3,3,3]; o=[1,2,3,4,5,6,7,8,9];
484 ii=[1,2;3,4;6,7;8,9];
485 iii=[1,3;2,4;6,8;7,9];
486 v=[1,6;2,7;3,8;4,9];

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487 in2=□ ;  
488 in3=□ ;  
489 in5=□ ;

## Bibliography

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